Exercise 19

Compute Δy and dy for the given values of x and $dx = \Delta x$. Then sketch a diagram like Figure 5 showing the line segments with lengths dx, dy, and Δy.

$$y = x^2 - 4x, \quad x = 3, \quad \Delta x = 0.5$$

Solution

Compute the derivative of y.

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 - 4x)$$

$$= 2x - 4$$

Consequently, the differential of $y = x^2 - 4x$ is

$$dy = (2x - 4)\,dx,$$

so when $x = 3$ and $\Delta x = dx = 0.5$,

$$dy = [2(3) - 4](0.5) = 1$$

$$\Delta y = y(3 + 0.5) - y(3) = [(3 + 0.5)^2 - 4(3 + 0.5)] - [(3)^2 - 4(3)] = 1.25.$$

The function is plotted below along with its tangent line at $x = 3$.